Neutrino telescopes as a probe of active and sterile neutrino mixings

Zhi-zhong Xing[a]

[a]Institute of High Energy Physics, P.O. Box 918, Beijing 100049, China

If the ultrahigh-energy (UHE) neutrino fluxes produced from a distant astrophysical source can be measured at a km^3-size neutrino telescope, they will provide a promising way to help determine the flavor mixing pattern of three active neutrinos. Considering the conventional UHE neutrino source with the flavor ratio $\phi_e : \phi_\mu : \phi_\tau = 1 : 2 : 0$, I show that $\phi_e^{\rm D} : \phi_\mu^{\rm D} : \phi_\tau^{\rm D} = (1 - 2\Delta) : (1 + \Delta) : (1 + \Delta)$ holds at the detector of a neutrino telescope, where Δ characterizes the effect of μ-τ symmetry breaking (i.e., $\theta_{13} \neq 0$ and $\theta_{23} \neq \pi/4$). Current experimental data yield $-0.1 \leq \Delta \leq +0.1$. It is also possible to probe Δ by detecting the $\overline{\nu}_e$ flux of $E_{\overline{\nu}_e} \approx 6.3$ PeV via the Glashow resonance channel $\overline{\nu}_e e \to W^- \to$ anything. Finally, I give some brief comments on the possibility to constrain the mixing between active and sterile neutrinos by using the UHE neutrino telescopes.

1. Introduction

The solar [1], atmospheric [2], reactor [3] and accelerator [4] neutrino experiments have provided convincing evidence for the existence of neutrino oscillations and opened a window to new physics beyond the standard model. The neutrino mixing is described by a unitary matrix V,

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} V_{e1} & V_{e2} & V_{e3} \\ V_{\mu1} & V_{\mu2} & V_{\mu3} \\ V_{\tau1} & V_{\tau2} & V_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} . \quad (1)$$

In the "standard" parametrization of V [5], one defines $V_{e2} = \sin\theta_{12} \cos\theta_{13}$, $V_{e3} = \sin\theta_{13} e^{-i\delta}$ and $V_{\mu3} = \sin\theta_{23} \cos\theta_{13}$. Here I have omitted the Majorana CP-violating phases from V, because they are irrelevant to the properties of neutrino oscillations to be discussed. A global analysis of current experimental data (see, e.g., Ref. [6]) points to $\theta_{13} = 0$ and $\theta_{23} = \pi/4$, which motivate a number of authors to consider the μ-τ permutation symmetry for model building [7].

The main purpose of my talk is to investigate how the effect of μ-τ symmetry breaking can show up at a neutrino telescope. I anticipate that Ice-Cube [8] and other second-generation neutrino telescopes [9] are able to detect the fluxes of ultrahigh-energy (UHE) ν_e ($\overline{\nu}_e$), ν_μ ($\overline{\nu}_\mu$) and ν_τ ($\overline{\nu}_\tau$) neutrinos generated from very distant astrophysical sources. For most of the currently-envisaged sources of UHE neutrinos [10], a general and canonical expectation is that the initial neutrino fluxes are produced via the decay of pions created from pp or $p\gamma$ collisions and their flavor content can be expressed as

$$\{\phi_e , \phi_\mu , \phi_\tau\} = \left\{\frac{1}{3} , \frac{2}{3} , 0\right\} \phi_0 , \quad (2)$$

where ϕ_α (for $\alpha = e, \mu, \tau$) denotes the sum of ν_α and $\overline{\nu}_\alpha$ fluxes, and $\phi_0 = \phi_e + \phi_\mu + \phi_\tau$ is the total flux of neutrinos and antineutrinos of all flavors. Due to neutrino oscillations, the flavor composition of such cosmic neutrino fluxes to be measured at the detector of a neutrino telescope has been expected to be [11]

$$\{\phi_e^{\rm D}, \phi_\mu^{\rm D}, \phi_\tau^{\rm D}\} = \left\{\frac{1}{3} , \frac{1}{3} , \frac{1}{3}\right\} \phi_0 . \quad (3)$$

However, it is worth remarking that this naive expectation is only true in the limit of μ-τ symmetry (or equivalently, $\theta_{13} = 0$ and $\theta_{23} = \pi/4$). Starting from the hypothesis given in Eq. (2) and allowing for the slight breaking of μ-τ symmetry, I am going to show that

$$\phi_e^{\rm D} : \phi_\mu^{\rm D} : \phi_\tau^{\rm D} = (1 - 2\Delta) : (1 + \Delta) : (1 + \Delta) \quad (4)$$

holds to an excellent degree of accuracy, where Δ characterizes the effect of μ-τ symmetry breaking (i.e., the combined effect of $\theta_{13} \neq 0$ and $\theta_{23} \neq \pi/4$) [12]. I obtain $-0.1 \leq \Delta \leq +0.1$ from the present neutrino oscillation data. I find that it is also possible to probe Δ by detecting the $\overline{\nu}_e$ flux of $E_{\overline{\nu}_e} \approx 6.3$ PeV via the

well-known Glashow resonance (GR) [13] channel $\overline{\nu}_e e \to W^- \to$ anything at a neutrino telescope. Finally, I will give some comments on the possibility to constrain the mixing between active and sterile neutrinos by using neutrino telescopes.

2. Signals of μ-τ symmetry breaking

Let me define $\phi_\alpha^{(D)} \equiv \phi_{\nu_\alpha}^{(D)} + \phi_{\overline{\nu}_\alpha}^{(D)}$ (for $\alpha = e, \mu, \tau$) throughout this paper, where $\phi_{\nu_\alpha}^{(D)}$ and $\phi_{\overline{\nu}_\alpha}^{(D)}$ denote the ν_α and $\overline{\nu}_\alpha$ fluxes, respectively. As for the UHE neutrino fluxes produced from the pion-muon decay chain with $\phi_{\nu_\tau} = \phi_{\overline{\nu}_\tau} = 0$, the relationship between ϕ_{ν_α} (or $\phi_{\overline{\nu}_\alpha}$) and $\phi_{\nu_\alpha}^D$ (or $\phi_{\nu_\alpha}^D$) is given by $\phi_{\nu_\alpha}^D = \phi_{\nu_e} P_{e\alpha} + \phi_{\nu_\mu} P_{\mu\alpha}$ or $\phi_{\overline{\nu}_\alpha}^D = \phi_{\overline{\nu}_e} \bar{P}_{e\alpha} + \phi_{\overline{\nu}_\mu} \bar{P}_{\mu\alpha}$, in which $P_{\beta\alpha}$ and $\bar{P}_{\beta\alpha}$ (for $\alpha = e, \mu, \tau$ and $\beta = e$ or μ) stand respectively for the oscillation probabilities $P(\nu_\beta \to \nu_\alpha)$ and $P(\overline{\nu}_\beta \to \overline{\nu}_\alpha)$. Because the Galactic distances far exceed the observed neutrino oscillation lengths, $P_{\beta\alpha}$ and $\bar{P}_{\beta\alpha}$ are actually averaged over many oscillations. Then I obtain $\bar{P}_{\beta\alpha} = P_{\beta\alpha}$ and

$$P_{\beta\alpha} = \sum_{i=1}^{3} |V_{\alpha i}|^2 |V_{\beta i}|^2 , \qquad (5)$$

where $V_{\alpha i}$ and $V_{\beta i}$ (for $\alpha, \beta = e, \mu, \tau$ and $i = 1, 2, 3$) denote the matrix elements of V defined in Eq. (1). The relationship between ϕ_α and ϕ_α^D turns out to be

$$\phi_\alpha^D = \phi_e P_{e\alpha} + \phi_\mu P_{\mu\alpha} . \qquad (6)$$

To be explicit, I have

$$\phi_e^D = \frac{\phi_0}{3} \left(P_{ee} + 2P_{\mu e} \right) ,$$

$$\phi_\mu^D = \frac{\phi_0}{3} \left(P_{e\mu} + 2P_{\mu\mu} \right) ,$$

$$\phi_\tau^D = \frac{\phi_0}{3} \left(P_{e\tau} + 2P_{\mu\tau} \right) . \qquad (7)$$

It is then possible to evaluate the relative sizes of ϕ_e^D, ϕ_μ^D and ϕ_τ^D by using Eqs. (1), (5) and (7).

In order to clearly show the effect of μ-τ symmetry breaking on the neutrino fluxes to be detected at neutrino telescopes, I define

$$\varepsilon \equiv \theta_{23} - \frac{\pi}{4} , \qquad (|\varepsilon| \ll 1) . \qquad (8)$$

Namely, ε measures the slight departure of θ_{23} from $\pi/4$. Using small θ_{13} and ε, I express $|V_{\alpha i}|^2$ (for $\alpha = e, \mu, \tau$ and $i = 1, 2, 3$) as follows:

$$\begin{aligned} \left[|V_{\alpha i}|^2 \right] &= \frac{1}{2} A + \varepsilon B + \frac{1}{2} \left(\theta_{13} \sin 2\theta_{12} \cos \delta \right) C \\ &\quad + \mathcal{O}(\varepsilon^2) + \mathcal{O}(\theta_{13}^2) . \end{aligned} \qquad (9)$$

where

$$A = \begin{bmatrix} 2\cos^2\theta_{12} & 2\sin^2\theta_{12} & 0 \\ \sin^2\theta_{12} & \cos^2\theta_{12} & 1 \\ \sin^2\theta_{12} & \cos^2\theta_{12} & 1 \end{bmatrix} ,$$

$$B = \begin{bmatrix} 0 & 0 & 0 \\ -\sin^2\theta_{12} & -\cos^2\theta_{12} & 1 \\ \sin^2\theta_{12} & \cos^2\theta_{12} & -1 \end{bmatrix} ,$$

$$C = \begin{bmatrix} 0 & 0 & 0 \\ 1 & -1 & 0 \\ -1 & 1 & 0 \end{bmatrix} .$$

Eqs. (5) and (9) allow me to calculate $P_{\beta\alpha}$:

$$\begin{aligned} P_{ee} + 2P_{\mu e} &= 1 + \frac{\theta_{13}}{2} \sin 4\theta_{12} \cos \delta \\ &\quad - \varepsilon \sin^2 2\theta_{12} , \\ P_{e\mu} + 2P_{\mu\mu} &= 1 - \frac{\theta_{13}}{4} \sin 4\theta_{12} \cos \delta \\ &\quad + \frac{\varepsilon}{2} \sin^2 2\theta_{12} , \\ P_{e\tau} + 2P_{\mu\tau} &= 1 - \frac{\theta_{13}}{4} \sin 4\theta_{12} \cos \delta \\ &\quad + \frac{\varepsilon}{2} \sin^2 2\theta_{12} , \end{aligned} \qquad (10)$$

where the terms of $\mathcal{O}(\varepsilon^2)$ and $\mathcal{O}(\theta_{13}^2)$ are omitted. Substituting Eq. (10) into Eq. (7), I get [12]

$$\phi_e^D = \frac{\phi_0}{3} \left(1 - 2\Delta \right) ,$$

$$\phi_\mu^D = \frac{\phi_0}{3} \left(1 + \Delta \right) ,$$

$$\phi_\tau^D = \frac{\phi_0}{3} \left(1 + \Delta \right) , \qquad (11)$$

where

$$\Delta = \frac{1}{4} \left(2\varepsilon \sin^2 2\theta_{12} - \theta_{13} \sin 4\theta_{12} \cos \delta \right) . \qquad (12)$$

Eq. (4) is therefore proved by Eq. (11). One can see that $\phi_e^D + \phi_\mu^D + \phi_\tau^D = \phi_0$ holds. Some discussions are in order.

(1) The small parameter Δ characterizes the overall effect of μ-τ symmetry breaking. Allowing δ to vary between 0 and π, I obtain the lower and upper bounds of Δ for given values of θ_{12} ($< \pi/4$), θ_{13} and ε: $-\Delta_{\mathrm{bound}} \leq \Delta \leq +\Delta_{\mathrm{bound}}$, where

$$\Delta_{\mathrm{bound}} = \frac{1}{4}\left(2|\varepsilon|\sin^2 2\theta_{12} + \theta_{13}\sin 4\theta_{12}\right) . \quad (13)$$

It is obvious that $\Delta = -\Delta_{\mathrm{bound}}$ when $\varepsilon < 0$ and $\delta = 0$, and $\Delta = +\Delta_{\mathrm{bound}}$ when $\varepsilon > 0$ and $\delta = \pi$. A global analysis of current neutrino oscillation data [6] indicates $30° < \theta_{12} < 38°$, $\theta_{13} < 10°$ (≈ 0.17) and $|\varepsilon| < 9°$ (≈ 0.16) at the 99% confidence level, but the CP-violating phase δ is entirely unrestricted. Using these constraints, I analyze the allowed range of Δ and its dependence on δ. The maximal value of Δ_{bound} (i.e., $\Delta_{\mathrm{bound}} \approx 0.098$) appears when $|\varepsilon|$ and θ_{13} approach their respective upper limits and $\theta_{12} \approx 33°$ holds [12]. Δ_{bound} is not very sensitive to the variation of θ_{12} in its allowed region.

If $\theta_{13} = 0$ holds, $\Delta_{\mathrm{bound}} = 0.5|\varepsilon|\sin^2 2\theta_{12} < 0.074$ when θ_{12} approaches its upper limit. If $\varepsilon = 0$ (i.e., $\theta_{23} = \pi/4$) holds, I obtain $\Delta_{\mathrm{bound}} = 0.25\theta_{13}\sin 4\theta_{12} < 0.038$ when θ_{12} approaches its lower limit. Thus Δ_{bound} is more sensitive to the deviation of θ_{23} from $\pi/4$.

(2) Of course, $\Delta = 0$ exactly holds when $\theta_{13} = \varepsilon = 0$ is taken. Because the sign of ε and the range of δ are both unknown, we are now unable to rule out the nontrivial possibility $\Delta \approx 0$ in the presence of $\theta_{13} \neq 0$ and $\varepsilon \neq 0$. In other words, Δ may be vanishing or extremely small if its two leading terms cancel each other. It is easy to arrive at $\Delta \approx 0$ from Eq. (12), if the condition

$$\frac{\varepsilon}{\theta_{13}} = \cot 2\theta_{12}\cos\delta \quad (14)$$

is satisfied. Because of $|\cos\delta| \leq 1$, Eq. (14) imposes a strong constraint on the magnitude of ε/θ_{13}. The dependence of ε/θ_{13} on δ is illustrated in Ref. [12], where θ_{12} varies in its allowed range. I find that $|\varepsilon|/\theta_{13} < 0.6$ is necessary to hold, such that a large cancellation between two leading terms of Δ is possible to take place.

The implication of the above result on UHE neutrino telescopes is two-fold. On the one hand, an observable signal of $\Delta \neq 0$ at a neutrino tele-

scope implies the existence of significant μ-τ symmetry breaking. If a signal of $\Delta \neq 0$ does not show up at a neutrino telescope, on the other hand, one cannot conclude that the μ-τ symmetry is an exact or almost exact symmetry. It is therefore meaningful to consider the complementarity between neutrino telescopes and terrestrial neutrino oscillation experiments [14], in order to finally pin down the parameters of neutrino mixing and leptonic CP violation.

(3) To illustrate, I define the flux ratios

$$\begin{aligned}
R_e &\equiv \frac{\phi_e^{\mathrm{D}}}{\phi_\mu^{\mathrm{D}} + \phi_\tau^{\mathrm{D}}} , \\
R_\mu &\equiv \frac{\phi_\mu^{\mathrm{D}}}{\phi_\tau^{\mathrm{D}} + \phi_e^{\mathrm{D}}} , \\
R_\tau &\equiv \frac{\phi_\tau^{\mathrm{D}}}{\phi_e^{\mathrm{D}} + \phi_\mu^{\mathrm{D}}} ,
\end{aligned} \quad (15)$$

which may serve as the *working* observables at neutrino telescopes [15]. At least, R_μ can be extracted from the ratio of muon tracks to showers at IceCube [8], even if those electron and tau events cannot be disentangled. Taking account of Eq. (11), I approximately obtain

$$\begin{aligned}
R_e &\approx \frac{1}{2} - \frac{3}{2}\Delta , \\
R_\mu &\approx \frac{1}{2} + \frac{3}{4}\Delta , \\
R_\tau &\approx \frac{1}{2} + \frac{3}{4}\Delta .
\end{aligned} \quad (16)$$

It turns out that R_e is most sensitive to the effect of μ-τ symmetry breaking.

Due to $\phi_\mu^{\mathrm{D}} = \phi_\tau^{\mathrm{D}}$ shown in Eq. (11), $R_\mu = R_\tau$ holds no matter whether Δ vanishes or not. This observation implies that the "μ-τ" symmetry between R_μ and R_τ is actually insensitive to the breaking of μ-τ symmetry in the neutrino mass matrix. If both R_e and R_μ are measured, one can then extract Δ from their difference:

$$R_\mu - R_e = \frac{9}{4}\Delta . \quad (17)$$

Taking $\Delta = \Delta_{\mathrm{bound}} \approx 0.1$, we get $R_\mu - R_e \leq 0.22$.

3. On the Glashow resonance

I proceed to discuss the possibility to probe the breaking of μ-τ symmetry by detecting the $\overline{\nu}_e$

flux from distant astrophysical sources through the so-called Glashow resonance (GR) channel $\overline{\nu}_e e \to W^- \to$ anything [13]. The latter can take place over a very narrow energy interval around the $\overline{\nu}_e$ energy $E_{\overline{\nu}_e}^{\mathrm{GR}} \approx M_W^2/2m_e \approx 6.3$ PeV. A neutrino telescope may measure both the GR-mediated $\overline{\nu}_e$ events ($N_{\overline{\nu}_e}^{\mathrm{GR}}$) and the $\nu_\mu + \overline{\nu}_\mu$ events of charged-current (CC) interactions ($N_{\nu_\mu+\overline{\nu}_\mu}^{\mathrm{CC}}$) in the vicinity of $E_{\overline{\nu}_e}^{\mathrm{GR}}$. Their ratio, defined as $R_{\mathrm{RG}} \equiv N_{\overline{\nu}_e}^{\mathrm{GR}}/N_{\nu_\mu+\overline{\nu}_\mu}^{\mathrm{CC}}$, can be related to the ratio of $\overline{\nu}_e$'s to ν_μ's and $\overline{\nu}_\mu$'s entering the detector,

$$R_0 \equiv \frac{\phi_{\overline{\nu}_e}^{\mathrm{D}}}{\phi_{\nu_\mu}^{\mathrm{D}} + \phi_{\overline{\nu}_\mu}^{\mathrm{D}}} \ . \tag{18}$$

Note that $\phi_{\overline{\nu}_e}^{\mathrm{D}}$, $\phi_{\nu_\mu}^{\mathrm{D}}$ and $\phi_{\overline{\nu}_\mu}^{\mathrm{D}}$ stand respectively for the fluxes of $\overline{\nu}_e$'s, ν_μ's and $\overline{\nu}_\mu$'s before the RG and CC interactions occur at the detector. In a recent paper [16], $R_{\mathrm{GR}} = aR_0$ with $a \approx 30.5$ has been obtained by considering the muon events with contained vertices [17] in a water- or ice-based detector. An accurate calculation of a is crucial for a specific neutrino telescope to detect the GR reaction rate, but it is beyond the scope of this talk. Here I only concentrate on the possible effect of μ-τ symmetry breaking on R_0.

Provided the initial neutrino fluxes are produced via the decay of π^+'s and π^-'s created from high-energy pp collisions, their flavor composition can be expressed in a more detailed way as

$$\left\{ \phi_{\nu_e}, \ \phi_{\nu_\mu}, \ \phi_{\nu_\tau} \right\} = \left\{ \frac{1}{6}, \ \frac{1}{3}, \ 0 \right\} \phi_0 \ ,$$
$$\left\{ \phi_{\overline{\nu}_e}, \ \phi_{\overline{\nu}_\mu}, \ \phi_{\overline{\nu}_\tau} \right\} = \left\{ \frac{1}{6}, \ \frac{1}{3}, \ 0 \right\} \phi_0 \ . \tag{19}$$

In comparison, the flavor content of UHE neutrino fluxes produced from $p\gamma$ collisions reads

$$\left\{ \phi_{\nu_e}, \ \phi_{\nu_\mu}, \ \phi_{\nu_\tau} \right\} = \left\{ \frac{1}{3}, \ \frac{1}{3}, \ 0 \right\} \phi_0 \ ,$$
$$\left\{ \phi_{\overline{\nu}_e}, \ \phi_{\overline{\nu}_\mu}, \ \phi_{\overline{\nu}_\tau} \right\} = \left\{ 0, \ \frac{1}{3}, \ 0 \right\} \phi_0 \ . \tag{20}$$

For either Eq. (19) or Eq. (20), the sum of ϕ_{ν_α} and $\phi_{\overline{\nu}_\alpha}$ is consistent with ϕ_α in Eq. (2).

Due to neutrino oscillations, the $\overline{\nu}_e$ flux at the detector of a neutrino telescope is given by $\phi_{\overline{\nu}_e}^{\mathrm{D}} =$

$\phi_{\overline{\nu}_e} \bar{P}_{ee} + \phi_{\overline{\nu}_\mu} \bar{P}_{\mu e}$. With the help of Eqs. (5), (9), (19) and (20), I explicitly obtain

$$\phi_{\overline{\nu}_e}^{\mathrm{D}}(pp) = \frac{\phi_0}{6}\left(1 - 2\Delta\right) \ ,$$
$$\phi_{\overline{\nu}_e}^{\mathrm{D}}(p\gamma) = \frac{\phi_0}{12}\left(\sin^2 2\theta_{12} - 4\Delta\right) \ . \tag{21}$$

The sum of $\phi_{\nu_\mu}^{\mathrm{D}}$ and $\phi_{\overline{\nu}_\mu}^{\mathrm{D}}$, which is defined as ϕ_μ^{D}, has been given in Eq. (11). It is then straightforward to calculate R_0 by using Eq. (18) for two different astrophysical sources:

$$R_0(pp) \approx \frac{1}{2} - \frac{3}{2}\Delta \ ,$$
$$R_0(p\gamma) \approx \frac{\sin^2 2\theta_{12}}{4} - \frac{4 + \sin^2 2\theta_{12}}{4}\Delta \ . \tag{22}$$

This result indicates that the dependence of $R_0(pp)$ on θ_{12} is hidden in Δ and suppressed by the smallness of θ_{13} and ε. In addition, the deviation of $R_0(pp)$ from $1/2$ can be as large as $1.5\Delta_{\mathrm{bound}} \approx 0.15$. It is obvious that the ratio $R_0(p\gamma)$ is very sensitive to the value of $\sin^2 2\theta_{12}$. A measurement of $R_0(p\gamma)$ at IceCube and other second-generation neutrino telescopes may therefore probe the mixing angle θ_{12} [16]. Indeed, the dominant production mechanism for ultrahigh-energy neutrinos at Active Galactic Nuclei (AGNs) and Gamma Ray Bursts (GRBs) is expected to be the $p\gamma$ process in a tenuous or radiation-dominated environment [18]. If this expectation is true, the observation of $R_0(p\gamma)$ may also provide us with useful information on the breaking of μ-τ symmetry.

4. Comments on sterile neutrinos

Today we are not well motivated to consider the existence of very light sterile neutrinos, which may take part in the oscillations of active neutrinos and change the conventional interpretation of current experimental results [6]. In particular, it is hard to simultaneously interpret the LSND anomaly [19] and other convincing neutrino oscillation data by introducing one or two sterile neutrinos. The mixing between sterile and active neutrinos has to be sufficiently suppressed; otherwise, it might bring sterile neutrinos in equilibrium with active neutrinos before neutrino de-

coupling — the resultant excess in energy dependence would endanger the Big Bang nucleosynthesis of light elements [20]. If the mass-squared differences between active and sterile neutrinos are of $\mathcal{O}(10^{-11})$ eV^2 or smaller, however, neutrino oscillations will not produce and maintain a significant sterile neutrino population. This case has been considered in Ref. [21] with a conclusion that the UHE neutrinos may offer a unique opportunity to probe neutrino oscillations in the mass-squared range 10^{-16} eV$^2 \leq \Delta m^2 \leq 10^{-11}$ eV^2, a region which is not accessible by any other means.

The possible effects of sterile neutrinos on the UHE neutrino fluxes have been discussed in the literature (see, e.g., Refs. [21] and [22]). For simplicity, I do not elaborate on them in this talk. I only emphasize that some of such discussions are already out of date, because the experimental constraints on the mixing between sterile and active neutrinos have become more stringent than before. Whether the light sterile neutrinos exist or not remains an open question.

5. Comments on the ratio $\phi_e : \phi_\mu : \phi_\tau$

What I have so far considered is the canonical or conventional astrophysical source, from which the UHE neutrino flux results from the pion decays and thus has the flavor composition $\phi_e : \phi_\mu : \phi_\tau = 1 : 2 : 0$. In reality, however, this simple flavor content could somehow by contaminated for certain reasons (e.g., a small amount of ν_e, ν_μ and ν_τ and their antiparticles might come from the decays of heavier hadrons produced by pp and $p\gamma$ collisions. Following a phenomenological approach, Zhou and I [15] proposed a generic parametrization of the initial flavor composition of an UHE neutrino flux:

$$\begin{pmatrix} \phi_e \\ \phi_\mu \\ \phi_\tau \end{pmatrix} = \begin{pmatrix} \sin^2\xi\cos^2\zeta \\ \cos^2\xi\cos^2\zeta \\ \sin^2\zeta \end{pmatrix} \phi_0 \,, \qquad (23)$$

where $\xi \in [0, \pi/2]$ and $\zeta \in [0, \pi/2]$. Then the conventional picture, as shown in Eq. (2), corresponds to $\zeta = 0$ and $\tan\xi = 1/\sqrt{2}$ (or $\xi \approx 35.3°$) in our parametrization. It turns out that any small departure of ζ from zero will measure the existence of cosmic ν_τ and $\overline{\nu}_\tau$ neutrinos, which

could come from the decays of D_s and $B\overline{B}$ mesons produced at the source [11]. On the other hand, any small deviation of $\tan^2\xi$ from $1/2$ will imply that the pure pion-decay mechanism for the UHE neutrino production has to be modified.

After defining three neutrino flux ratios R_α (see Eq. (15) for $\alpha = e, \mu, \tau$) as our working observables at a neutrino telescope, we have shown that the source parameters ξ and ζ can in principle be determined by the measurement of two independent R_α and with the help of accurate neutrino oscillation data [15]. We have also examined the dependence of R_α upon the smallest neutrino mixing angle θ_{13} and upon the Dirac CP-violating phase δ. Our numerical examples indicate that it is promising to determine or (at least) constrain the initial flavor content of UHE neutrino fluxes with the second-generation neutrino telescopes.

6. Concluding remarks

I have discussed why and how the second-generation neutrino telescopes can serve as a striking probe of broken μ-τ symmetry. Based on the conventional mechanism for UHE neutrino production at a distant astrophysical source and the standard picture of neutrino oscillations, I have shown that the flavor composition of cosmic neutrino fluxes at a terrestrial detector may deviate from the naive expectation $\phi_e^{\mathrm{D}} : \phi_\mu^{\mathrm{D}} : \phi_\tau^{\mathrm{D}} = 1 : 1 : 1$. Instead, $\phi_e^{\mathrm{D}} : \phi_\mu^{\mathrm{D}} : \phi_\tau^{\mathrm{D}} = (1 - 2\Delta) : (1 + \Delta) : (1 + \Delta)$ holds, where Δ characterizes the effect of μ-τ symmetry breaking. The latter is actually a reflection of $\theta_{13} \neq 0$ and $\theta_{23} \neq \pi/4$ in the 3×3 neutrino mixing matrix. I have examined the sensitivity of Δ to the deviation of θ_{13} from zero and to the departure of θ_{23} from $\pi/4$, and obtained $-0.1 \leq \Delta \leq +0.1$ from current data. I find that it is also possible to probe the breaking of μ-τ symmetry by detecting the $\overline{\nu}_e$ flux of $E_{\overline{\nu}_e} \approx 6.3$ PeV via the Glashow resonance channel $\overline{\nu}_e e \to W^- \to$ anything.

This work, different from the previous ones (see Refs. [14,15,16,23]) in studying how to determine or constrain one or two of three neutrino mixing angles and the Dirac CP-violating phase with neutrino telescopes, reveals the combined effect of $\theta_{13} \neq 0$, $\theta_{23} \neq \pi/4$ and $\delta \neq \pi/2$ which can

show up at the detector. Even if $\Delta \neq 0$ is established from the measurement of UHE neutrino fluxes, the understanding of this μ-τ symmetry breaking signal requires more precise information about θ_{13}, θ_{23} and δ. Hence it makes sense to look at the complementary roles played by neutrino telescopes and terrestrial neutrino oscillation experiments (e.g., the reactor experiments to pin down the smallest neutrino mixing angle θ_{13} and the neutrino factories or superbeam facilities to measure the CP-violating phase δ) in the era of precision measurements.

The feasibility of the above idea depends on the assumption that we have correctly understood the production mechanism of cosmic neutrinos from a distant astrophysical source (i.e., via pp and $p\gamma$ collisions) with little uncertainties. It is also dependent upon the assumption that the error bars associated with the measurement of relevant neutrino fluxes or their ratios are much smaller than Δ. The latter is certainly a challenge to the sensitivity or precision of IceCube and other neutrino telescopes under construction or under consideration, unless the effect of μ-τ symmetry breaking is unexpectedly large. Nevertheless, any constraint on Δ to be obtained from neutrino telescopes will be greatly useful in diagnosing the astrophysical sources and in understanding the properties of neutrinos themselves. Much more effort is therefore needed in this direction.

Finally, I would like to thank Y.Q. Ma and other organizers for kind invitation and warm hospitality. The symposium is as wonderful as the beach in Wei Hai, a fantastic place which is suitable for talking about the fantastic idea on neutrino telescopes. I am also grateful to Z. Cao and S. Zhou for many stimulating discussions on UHE cosmic rays and neutrino astronomy.

REFERENCES

1. SNO Collaboration, Q.R. Ahmad *et al.*, Phys. Rev. Lett. **89**, 011301 (2002).
2. For a review, see: C.K. Jung *et al.*, Ann. Rev. Nucl. Part. Sci. **51**, 451 (2001).
3. KamLAND Collaboration, K. Eguchi *et al.*, Phys. Rev. Lett. **90**, 021802 (2003).
4. K2K Collaboration, M.H. Ahn *et al.*, Phys. Rev. Lett. **90**, 041801 (2003).
5. Particle Data Group, S. Eidelman *et al.*, Phys. Lett. B **592**, 1 (2004); Z.Z. Xing, Int. J. Mod. Phys. A **19**, 1 (2004).
6. A. Strumia and F. Vissani, hep-ph/0606054.
7. Z.Z. Xing, H. Zhang, and S. Zhou, Phys. Lett. B **641**, 189 (2006); and references therein.
8. J. Ahrens *et al.* (IceCube), Nucl. Phys. (Proc. Suppl.) **118**, 388 (2003).
9. E. Aslanides *et al.* (ANTARES), astro-ph/9907432; S.E. Tzamarias (NESTOR), Nucl. Instrum. Meth. A **502**, 150 (2003); P. Piattelli (NEMO), Nucl. Phys. (Proc. Suppl.) **143**, 359 (2005).
10. J.G. Learned and K. Mannheim, Annu. Rev. Nucl. Part. Sci. **50**, 679 (2000); F. Halzen and D. Hooper, Rept. Prog. Phys. **65**, 1025 (2002).
11. J.G. Learned and S. Pakvasa, Astropart. Phys. **3**, 267 (1995).
12. Z.Z. Xing, Phys. Rev. D **74**, 013009 (2006).
13. S.L. Glashow, Phys. Rev. **118**, 316 (1960).
14. W. Winter, Phys. Rev. D **74**, 033015 (2006).
15. Z.Z. Xing and S. Zhou, Phys. Rev. D **74**, 013010 (2006).
16. P. Bhattacharjee and N. Gupta, hep-ph/0501191.
17. J.F. Beacom *et al.*, Phys. Rev. D **68**, 093005 (2003); Erratum-*ibid.* D **72**, 019901 (2005).
18. V.S. Berezinsky *et al.*, *Astrophysics of Cosmic Rays* (North-Holland, Amsterdam, 1990); T.K. Gaisser, F. Halzen, and T. Stanev, Phys. Rept. **258**, 173 (1995).
19. LSND Collaboration, A. Aguilar *et al.*, Phys. Rev. D **64**, 112007 (2001).
20. R. Barbieri and A. Dolgov, Phys. Lett. B **237**, 440 (1990); Nucl. Phys. B **349**, 742 (1991).
21. P. Keränen *et al.*, Phys. Lett. B **574**, 162 (2003).
22. H. Athar, M. Jezabek, and O. Yasuda, Phys. Rev. D **62**, 103007 (2000).
23. G.J. Gounaris and G. Moultaka, hep-ph/0212110; G. Barenboim and C. Quigg, Phys. Rev. D **67**, 073024 (2003); J.F. Beacom *et al.*, Phys. Rev. D **69**, 017303 (2004); P.D. Serpico and M. Kachelrieß, Phys. Rev. Lett. **94**, 211102 (2005); P.D. Serpico, Phys. Rev. D **73**, 047301 (2006).